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Anderson transitions in a random magnetic field

Tohru Kawarabayashi†, Bernhard Kramer‡ and Tomi Ohtsuki§

† Institute for Solid State Physics, University of Tokyo, Roppongi 7-22-1, Minato-ku, Tokyo 106-8666, Japan

‡ Institut für Theoretische Physik, Universität Hamburg, Jungiusstraße 9, D-20355 Hamburg, Germany

§ Department of Physics, Sophia University, Kioi-cho 7-1, Chiyoda-ku, Tokyo 102-8554, Japan

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Abstract. The Anderson transitions in a random magnetic field in three dimensions are investigated in detail by means of the transfer-matrix method with high accuracy. Both systems with and systems without an additional random scalar potential are considered. We find the critical exponent ν for the localization length to be 1.45 ± 0.09 with a strong random scalar potential. Without it, the exponent is smaller but increases with the system size and extrapolates to the above value within the error bars. These results support the conventional classification of universality classes according to symmetry. The mobility edge trajectory in the random magnetic field is also obtained.

The metal–insulator transition driven by disorder, which is called the Anderson transition (AT), has attracted much attention for many years [1, 2]. The critical behaviour of the AT is conventionally classified, according to the symmetry of the Hamiltonians, into three universality classes: the orthogonal, the unitary and the symplectic classes. Systems invariant under spin rotation as well as time reversal form the orthogonal class. The unitary class is characterized by the absence of time-reversal symmetry. Systems invariant under time reversal but having no spin-rotation symmetry belong to the symplectic class.

The AT in a magnetic field has been studied extensively, mainly in connection with the quantum Hall effect [3]. The magnetic field breaks the time-reversal symmetry and thus all systems under a magnetic field should belong to the unitary class. In fact, it has been demonstrated numerically in three dimensions (3D) [4] that the critical behaviour is not sensitive to the strength of a uniform magnetic field. It has been pointed out, however, that in 3D the AT driven solely by a random vector potential might exhibit critical behaviour different from that observed in other unitary systems, for example systems having an additional random scalar potential [5]. Apparently, this questions the validity of the conventional classification of universality classes in AT. It is thus important to examine the critical behaviour in both cases with higher accuracy in order to clarify the validity of the unitary universality class in AT.

The AT in a random magnetic field is driven by the coherent scattering due to a fluctuating vector potential. A nontrivial feature of this coherent scattering by a fluctuating vector potential has been pointed out [6] in a theory of strongly correlated spin systems. Much work has also been done on transport properties in 2D in a random magnetic field, in particular in connection with the theory of the fractional quantum Hall effect [7] in a

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high magnetic field. It is thus an important issue to understand how the effect of coherent scattering in a strongly fluctuating random vector potential will show up in the AT.

In the present paper, we investigate the AT in 3D in a random magnetic field by means of the transfer-matrix method with considerably higher accuracy. In order to see the influence of the random scalar potential, we consider systems both with and without an additional random potential. High-accuracy analyses of the AT have been performed by several authors [8, 9]. In particular, in reference [9] the critical exponent has been accurately estimated for a system in a uniform magnetic field with a random scalar potential.

The model that we consider is described by the Hamiltonian

$$H = t \sum_{\langle i,j \rangle} \exp(\mathrm{i}\theta_{i,j}) C_i^\dagger C_j + \sum_i V_i C_i^\dagger C_i \tag{1}$$

where $C_i^\dagger$ (C_i) denotes a creation (annihilation) operator of an electron at the site i of a 3D cubic lattice. The energies $\{V_i\}$ are distributed independently and uniformly in the range $[-W/2, W/2]$. The Peierls phase factors $\exp(\mathrm{i}\theta_{i,j})$ describe a random magnetic field. We confine ourselves to considering the phases $\{\theta_{i,j}\}$ which are distributed independently and uniformly in the range $[-\pi, \pi]$. The hopping amplitude t is assumed to be the energy unit, $t = 1$.

We have performed the standard transfer-matrix method [8, 10] with high accuracy and evaluated the localization length ξ_M for the quasi-1D system with cross section $M \times M$ [11]. We assume the one-parameter scaling form $\Lambda_M = f(\xi/M)$ for $\Lambda_M \equiv \xi_M/M$, where ξ denotes the localization length at $M = \infty$. Since ξ diverges as $\xi \sim \delta x^{-\nu}$, the scaling function can be expanded as a function of δx as

$$\Lambda_M = \Lambda_c + \sum_{n=1}^{\infty} a_n (M^{1/\nu}\,\delta x)^n. \tag{2}$$

Here the variable δx measures the distance from the critical point; that is, $\delta x = (E - E_c)/E_c$ or $(W - W_c)/W_c$. In practice, we truncate this series at $n = 3$. By fitting the scaling function to the numerical data, we estimate the exponent ν and the critical point E_c or W_c.

For the transition at the band centre, a clear scaling has been observed for currently achievable sizes. We have estimated the critical disorder and the exponent ν to be $W_c = 18.80 \pm 0.04$ and $\nu = 1.45 \pm 0.09$ [11]. The renormalized localization length Λ_c at the critical point is 0.558 ± 0.003.

In contrast, for $W = 0$ or for an additional weak random scalar potential ($W = 1$), the critical point lies near the band edge, where the density of states changes rapidly as a function of energy. Through a careful analysis of the numerical data near the critical point for $W = 0$ and $W = 1$, we have found [11] that the correction to the scaling is not negligible for the transitions near the band edge.

Here we show, in table 1, a summary of the results for $W = 0$ obtained by means of fittings with different sizes including larger system sizes up to $M = 16$. The relative accuracy in ξ_M^{-1} achieved for $M = 14$ and $M = 16$ is 1% for each sample, and seven and five realizations of random phases are considered, respectively. The scaling regime is assumed to be the same as in reference [11]. It is clear that a critical point E_c exists at around 4.41 (see figure 1). The exponent ν tends to increase with the system size and is likely to saturate for $\nu \sim 1.48$. This size dependence, which is due to finite-size correction to the scaling, has also been observed for $W = 1$ [11]. Within the error bars, the estimated values of ν for $M \geqslant 12$ are consistent with 1.45 ± 0.09 obtained for the band centre as well as 1.43 ± 0.06 estimated for in the uniform magnetic field [9]. No evidence has been found for $\nu \approx 1$ which was suggested by calculations with low accuracy [5]. The present results

Table 1. Results for Λ_c, ν and E_c obtained by means of fits using the data for two system sizes M_1 and M_2 for $W = 0$. Here Q stands for the standard Q-value used in the χ^2-fit [14].

(M_1, M_2)	Λ_c	ν	E_c	Q
(6, 8)	0.514 ± 0.005	1.05 ± 0.07	4.414 ± 0.001	$\sim 10^{-5}$
(8, 10)	0.516 ± 0.007	1.26 ± 0.09	4.414 ± 0.001	~ 0.89
(10, 12)	0.51 ± 0.01	1.32 ± 0.12	4.414 ± 0.001	~ 0.99
(12, 14)	0.56 ± 0.02	1.49 ± 0.16	4.409 ± 0.002	~ 0.99
(14, 16)	0.48 ± 0.02	1.475 ± 0.19	4.417 ± 0.002	~ 0.61
(12, 16)	0.53 ± 0.01	1.46 ± 0.09	4.413 ± 0.001	~ 0.99

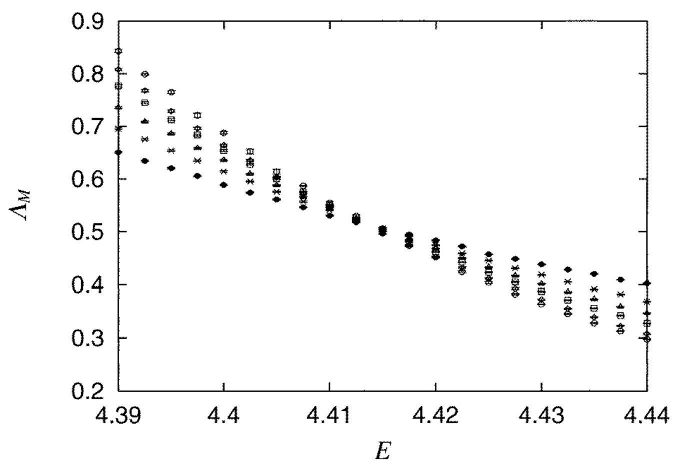

Figure 1. The renormalized localization length for $W = 0$ as a function of energy. The dots, the crosses, the triangles, the squares, the diamonds and the circles correspond to $M = 6$, 8, 10, 12, 14 and 16, respectively.

support the universality of the critical exponent in the unitary systems. The positions of the critical points and the values of Λ_c estimated with different combinations of system sizes are fluctuating for $M \geqslant 12$ (table 1). The value of $\Lambda_c = 0.558 \pm 0.003$ at the band centre seems to lie inside the range of this fluctuation. Conventionally, the value of Λ_c is also expected to be universal in unitary systems. Our results obtained here seem to be consistent with this universality of Λ_c.

We also estimate the critical points for various values of the energy E and disorder W (figure 2). The critical points (mobility edges) are estimated on the basis of numerical data by the transfer-matrix method with $M = 6$–10. It should be noted that there exist extended states for energies larger than the critical energy $E_c \approx 4.41$ for $W = 0$. This type of re-entrant phenomenon in the energy–disorder plane has been commonly observed for systems with a uniform distribution of a random scalar potential [12, 13]. It is interpreted [12] as indicating that the enhancement of extended states for a weak additional random scalar potential is due to the enhancement of the density of states for that energy regime.

In summary, we have investigated the AT in a random magnetic field numerically. By performing the transfer-matrix method with high accuracy, we have found that the correction to the scaling is not negligible for the currently achievable sizes for the transitions near the band edge. The exponents estimated for $W = 0$ for larger system sizes are consistent with

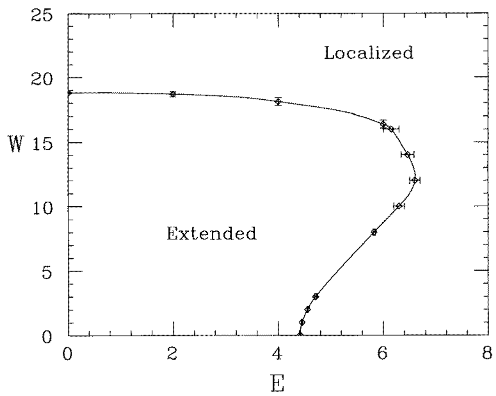

Figure 2. The mobility edge trajectory in 3D in the random magnetic field.

those obtained for other unitary systems within the error bars. From the size dependence of ν, in contrast to the suggestion in reference [5], no evidence has been found for $\nu \approx 1$. The mobility edge trajectory has also been obtained in the presence of a random magnetic field. Its qualitative shape turns out to be similar to those obtained for other systems with a uniform distribution of a random scalar potential.

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References

[1] Anderson P W 1958 *Phys. Rev.* **109** 1492
[2] Kramer B and MacKinnon A 1993 *Rep. Prog. Phys.* **56** 1469
[3] Hajdu J (ed) 1994 *Introduction to the Theory of the Integer Quantum Hall Effect* (Weinheim: VCH)
[4] Henneke M, Kramer B and Ohtsuki T 1994 *Eur. Phys. Lett.* **27** 389
[5] Ohtsuki T, Ono Y and Kramer B 1994 *J. Phys. Soc. Japan* **63** 685
[6] Altshuler B and Ioffe L B 1992 *Phys. Rev. Lett.* **69** 2979
[7] Halperin B I, Lee P A and Read N 1993 *Phys. Rev.* B **47** 7312
[8] MacKinnon A 1994 *J. Phys. C: Solid State Phys.* **6** 2511
[9] Slevin K and Ohtsuki T 1997 *Phys. Rev. Lett.* **78** 4083
[10] MacKinnon A and Kramer B 1981 *Phys. Rev. Lett.* **47** 1546
MacKinnon A and Kramer B 1983 *Z. Phys.* B **53** 1
[11] Kawarabayashi T, Kramer B and Ohtsuki T 1998 *Phys. Rev.* B **57** 11 842
[12] Bulka B, Schreiber M and Kramer B 1987 *Z. Phys.* B **66** 21
[13] Dröse T, Batsch M, Zharekeshev I and Kramer B 1998 *Phys. Rev.* B **57** 37
[14] Press W H, Teukolsky S A, Vetterling W T and Flannery B P 1992 *Numerical Recipes* 2nd edn (Cambridge: Cambridge University Press)